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The space-time pattern of price waves*

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Abstract. We investigate the way price fluctuations are transmitted between spatially separated markets. More specifically we show that the correlation patterns of wheat prices exhibit definite regularities some of which appear to be at variance with intuitive reasoning. Such patterns can be explained in the framework of a wave propagation model based on the so-called spatial arbitrage assumption. In 19th century France the velocity of price waves was of the order of 100 km/month. The economic implications of such an order of magnitude are discussed. In the concluding section we emphasize that what gives this problem its importance is its relative "simplicity", a word for which we propose an operational definition.

PACS. 01.75.+m Science and Society – 46.30.My Vibrations, mechanical waves and shocks – 47.35.+i Hydrodynamic waves – 89.90.+n Other areas of general interest to physicists

1 Introduction

In economics there are usually more answers than there are questions; or to say it in another way there are few really challenging questions for which no plausible explanations could be put forward. For example in order to "explain" a sudden peak in the price of a commodity a variety of mechanisms can be invoked: small yields due to bad weather conditions, wage increase for plantation workers, dwindling stocks, *etc.* In this paper we consider a question for which no straightforward explanation seems to be available. More specifically the paper shows that price fluctuations in a set of spatially separated markets display *spatial* as well as time-dependent patterns and that some of them are at variance with intuitive reasoning. Secondly it will be seen that by describing the price fluctuations by a (stochastic) wave equation one can account for the observed regularities; this justifies the expression "price waves" that we used in the title.

1.1 The challenge

First of all let us present the issue which challenged our curiosity. The observation concerns the behavior of wheat prices on a number of different markets; the main results are summarized in Figures 1a and 1b. Figure 1a shows the intercorrelation of (the logarithms of) prices between pairs of markets. The sample contains 11 markets, therefore there are $11 \times 10/2 = 55$ market pairs; each pair corresponds to one point. Not surprisingly the correlation decreases with distance; although the specific rate of that decrease would certainly require an explanation, at least the overall trend is consistent with the intuitive idea that more distant markets are less strongly interconnected. One could list a variety of reasons for such a behavior. Figure 1b is completely similar to Figure 1a except for the fact that there is a 6-month lag between the time series. The intriguing observation is of course that the correlation is now increasing rather than decreasing. This is not the only surprising observation as will be shown in Section 2. Before coming to this, however, we would like to look at the problem in a wider context.

1.2 Why should we study 19th century wheat prices?

When asked why he was claiming mountains Sir E. Hillary, the conqueror of the Everest, is reported to have answered "Because they are here". We could answer the question raised in the title of this paragraph in a similar vein. As a matter of fact in the natural sciences (physics, biology) any phenomenon that displays definite empirical regularities is deemed to deserve a thorough investigation. Yet in economics the tradition is somewhat different and Hillary's answer would hardly prove adequate. A clue for our interest in this problem is its "simplicity". We try to define that word more precisely in Appendix B. As far as wheat prices are concerned let us remind the following points:

(i) Before 1870-1875 wheat was by far the most important good both in terms of trade and in terms of consumption.
(ii) The prices of other cereals were closely correlated with the prices of wheat (see in this respect [9]).

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* The paper was written while the author was spending the fall term with the Dept of Economics at Harvard University (Cambridge, MA).

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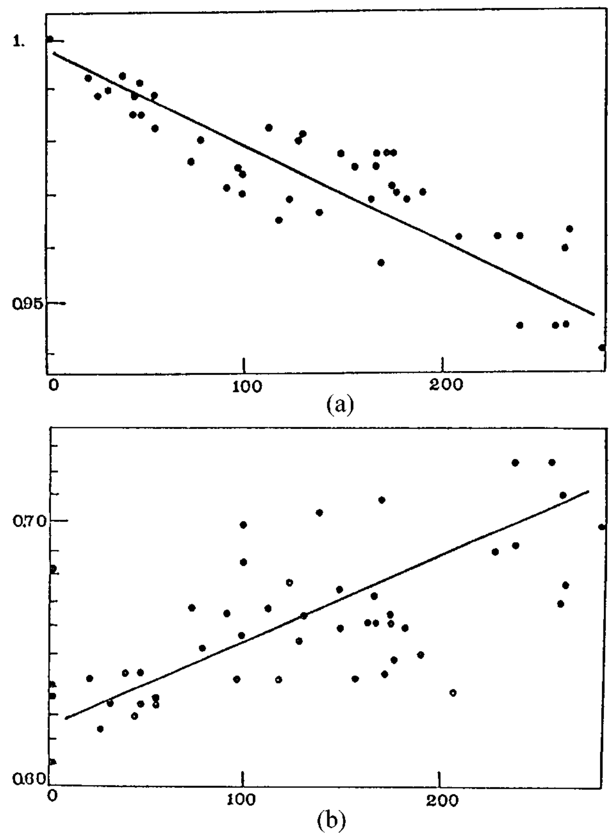

Fig. 1. (a) Decrease of wheat price correlation with distance (zero time-lag). Horizontal scale: kilometers. The sample contains 11 markets located in the center of France. The series are fortnight prices for the period 1841–1858. The coefficient of linear correlation is equal to −0.88 (confidence interval at probability 0.95: −0.79 to −0.94). Source for the data: [4]. (b) Increase of wheat price correlation with distance (8-month lag). Horizontal scale: kilometers. The sample of markets is the same as in the previous figure. The correlation is equal to 0.55 (confidence interval: 0.33 to 0.71). Source for the data: [4].

(iii) In a country such as 19th century France wheat exports or imports represented less than 5 percent of total production (see in this respect [4], p. 18]); therefore the domestic market was only loosely connected with foreign markets.

In short, because of its central importance in the economy the wheat market was only marginally affected by shocks originating from other sectors, and because it was largely self-sufficient it was also to a large extent isolated from foreign disturbances. These circumstances explain why the wheat market can be treated as an (almost) isolated system.

2 The space-time pattern of wheat prices

For this study one needs a set of price series providing a coverage of both time and space, that is to say prices recorded on several market places distributed more or less

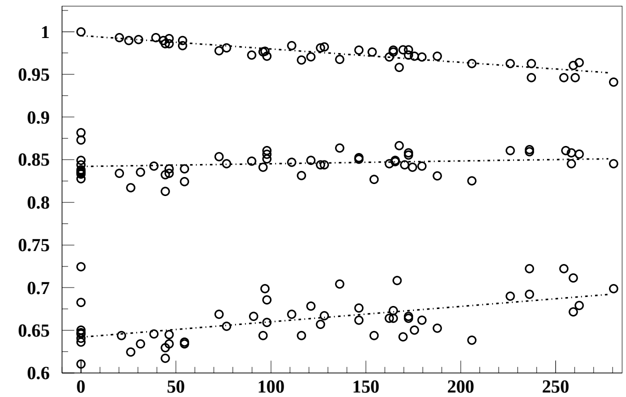

Fig. 2. Space-time correlation as a function of distance (horizontal scale in kilometers) for different time-lags. From top to bottom the time lags are: 0 month, 4 months, 8 months; the corresponding coefficient of correlation are: −0.88, 0.31, 0.55; the corresponding slopes expressed in 0.01/100 km are −1.6, 0.32, 1.8. The chart only shows the section of the curves for short distances; for sufficiently large inter-market distances the intercorrelations vanish of course. Source for the data: [4].

uniformly over the whole country, and for a time span of at least 50 years.

2.1 The data

In 19th century France there were about 500 market places for which prices were recorded without interruption from 1825 to 1913. This gives the possibility to study the space-time properties of the price field in a detailed way. There are comparable data (although with a smaller number of markets) for Bavaria and Prussia. Similar data also exist for the 20th century; for instance in the United States wheat prices are recorded on a state-level basis since the 1880s; those data are published annually by the USDA (United States Dept of Agriculture: Agricultural Statistics). It has been shown [11] that to a large extent the 20th century American wheat markets display the same features than the French 19th century markets. One drawback of the American data is the fact that the prices are state averages; therefore the location of the market is poorly defined especially for the largest states such as Texas, Oregon, Montana or Idaho.

In the following paragraphs we make a detailed study of inter-market correlations. It is true that for farmers and traders the meaningful variables are the *price differentials*; these have been analysed in [11] in an equilibrium framework. Here, however, our objective is to investigate the dynamic aspects of price fluctuations and for that purpose the intercorrelations are the adequate tools.

2.2 Price intercorrelations as a function of distance

Figure 2 shows the correlation $\rho_t(x)$ as a function of inter-market distance expressed in kilometers. As in Figure 1 each dot corresponds to a given market-pair. The three sets of dots respectively correspond to different time lags t ranging from 0 to 8 months (the first and the last have already been represented with different vertical scales in Figs. 1a and 1b). For $t = 0$ the slope s of the regression line is negative; for positive values of t it becomes positive and then increases to about 0.018 per 100 km for a 8-month lag.

2.3 Price intercorrelations as a function of time-lag

Figures 3a, 3b and 3c show the correlation $\rho_x(t)$ as a function of the time-lag t for different market pairs. Figure 3a is for markets which are about 100 kilometers apart, Figure 3b for markets about 400 kilometers apart and Figure 3c for markets about 700 kilometers apart. The downward trend of the curves was of course expected; more surprising is the occurrence of a plateau which becomes larger and larger as the distance between the markets increases. Such a feature would be difficult to explain by intuitive reasoning. In the following section we present a mathematical model which displays the observed properties.

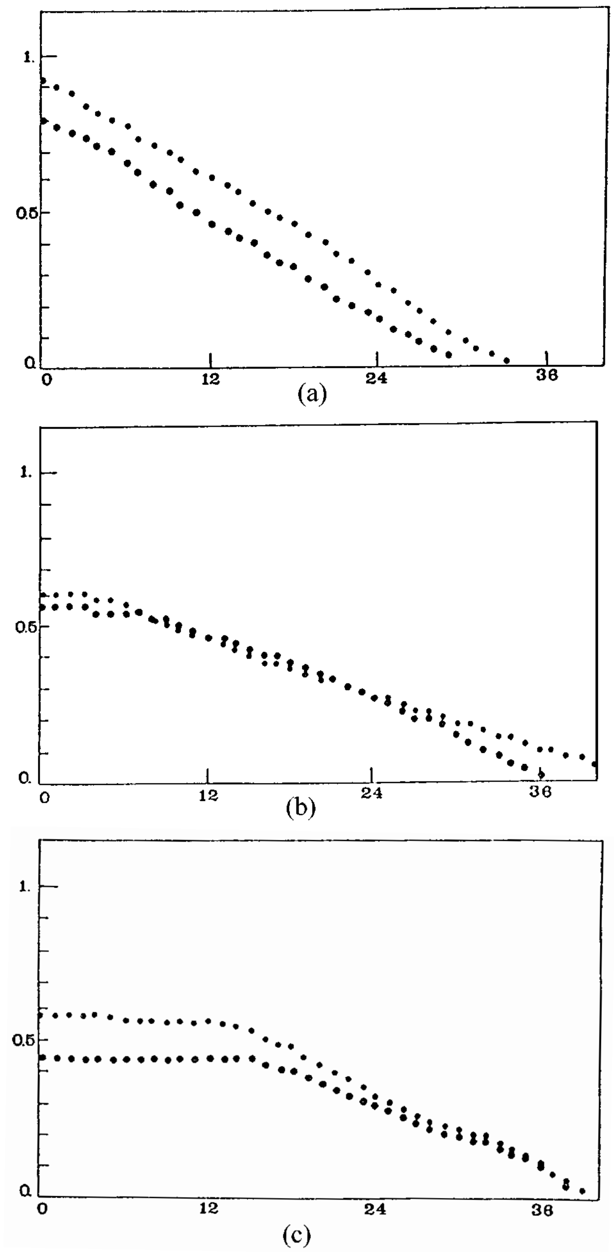

Fig. 3. (a) Space-time correlation as a function of the time-lag for two different pairs of markets. Horizontal scale: fortnights. Small dots: Albi-Montauban (64 km) 1825–1841; large dots: Mende-Albi (120 km) 1825-1841. (b) Space-time correlation as a function of the time-lag for two different pairs of markets. Horizontal scale: fortnights. Small dots: Beaugency-Mende (390 km) 1833–1850; large dots: Clermont-Pau (390 km) 1825-1841. (c) Space-time correlation as a function of the time-lag for two different pairs of markets. Horizontal scale: fortnights. Small dots: Douai-Montauban (720 km) 1830–1846; large dots: Arras-Albi (710 km) 1830–1846. Source for the data: [4]

3 The space-time pattern of random waves

In this section the emphasis is on underlying assumptions and on significant properties of the solutions; the mathematical framework is briefly reviewed in Appendix A.

3.1 Underlying assumptions

How do markets interact? This is the first question that has to be addressed. An answer is provided by the spatial arbitrage assumption that has been formalized by Samuelson in the 1950s [13]. Two markets do not interact so long as their price difference is smaller than the cost of transport. Once the price gap becomes larger than the transport cost, wheat is shipped from the market with the lowest price to the market with the highest price. This mechanism is summarized diagrammatically in Figure 4. The spatial arbitrage assumption has been at the basis of the so-called spatial price equilibrium model which was in vogue in the 1970s; see in this respect two very readable books [1,5].

Even with such a seemingly simple assumption for the interaction between markets the mathematical solution of of the problem for a set of N markets is by no means straightforward. Samuelson [13] was able to show that by using a variational formulation the problem can be reduced to the so-called transportation problem in linear programming, a problem which is itself of some complexity however. Here we use a different approach leading to analytical solutions.

3.2 The field equation

Once translated into a stochastic framework the spatial arbitrage assumption leads to the so-called SERS model [10]. Instead of considering a finite set of markets as would be natural, we write the SERS model for an infinite and

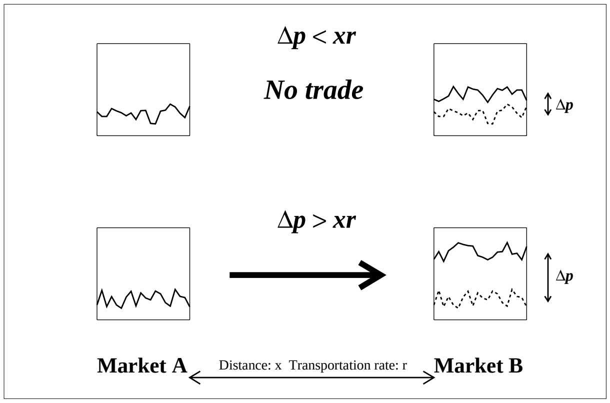

Fig. 4. Illustration of the spatial arbitrage assumption. For trade to take place between markets A and B the average price differential Δp between two markets has to be larger than current transport costs. In that case the quantity of wheat shipped from A to B is determined by the excess-supply functions for each market. Two circumstances contribute to complicate the situation: (1) transport costs strongly fluctuate in the course of time and are not statistically well-known. (ii) Trade reacts to the price gap with a substantial (but fairly unknown) inertia.

continuous set of markets; such a shift makes the model mathematically tractable.

3.2.1 The field equation and its solution

The stochastic price field $p(x,t)$ obeys the following stochastic partial differential equation:

$$Lp(x,t) = N(x,t) \tag{1}$$

where the linear differential operator L is defined through:

$$L(\partial_x, \partial_t) = (1/c^2)\partial^2_{t^2} + 2b\partial_t + a^2 - \partial^2_{x^2}$$

and where $N(x,t)$ represents Gaussian random disturbances.

The parameters a, b, c are expressed in the following units of measure:

$$[a] = 1/[L] \qquad [b] = [T]/[L^2] \qquad [c] = [L]/[T].$$

In terms of dimensional analysis c is similar to a velocity; in fact the determistic analogue of equation (1) is known [6] to describe the propagation of a wave in a dispersive medium (dispersive means that waves with different frequencies travel with somewhat different velocities). In what follows c will be considered as an estimate of the wave velocity.

In this paper we are interested in the space-time correlation function $\rho(x,t)$ that is to say the correlation between the prices $p(x',t')$ and the prices $p(x'+x,t'+t)$ at a point shifted by the distance x and the lag t. The correlation function resulting from the mathematical solution of equation (1) is given in Appendix A. Here we shall mainly focus on its qualitative features. One of these

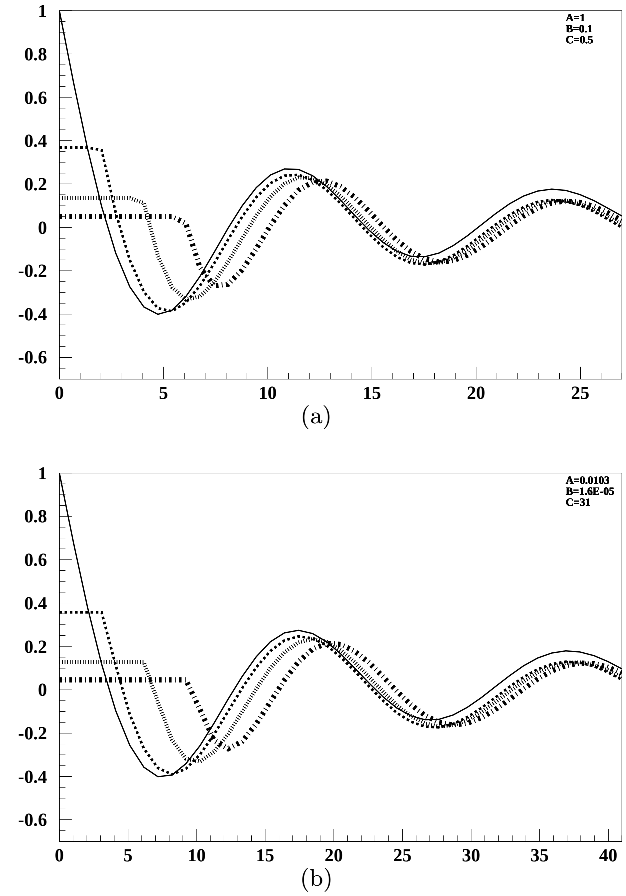

Fig. 5. Space-time correlation function as a function of time-lag for four different distances on each chart. Horizontal scale: time lag. (a) corresponds to the parameters: $a = 1$, $b = 0.1$, $c = 0.5$. From top to bottom the curves correspond to distances equal to 0, 1, 2, 3, (b) corresponds to $a = 0.0103$, $b = 6\times10^{-6}$, $c = 31$. From top to bottom the curves correspond to distances equal to 0, 100, 200, 300. The figure provides a verification of the scaling property; because the space and time variables have been rescaled, the curves have the same shape in spite of the fact that the parameter sets are different.

is the scale invariance property which can be formulated in the following terms:

If the distances are changed by a scale factor λ and if the time scale is divided by a factor θ, *i.e.*: $x = \lambda x' \quad t = \theta t'$, then the solution $\rho(x,t;a,b,c)$ is changed into $\rho(x',t';a',b',c')$ where:

$$a' = a\lambda, \quad b' = b\lambda^2/\theta, \quad c' = c\theta/\lambda.$$

That statement is illustrated in Figures 5a and 5b. As can be seen the curves for the correlation $\rho_x(t)$ are identical on both figures in spite of the fact that they correspond to different sets of parameters, namely: $a = 1$, $b = 0.1$, $c = 0.1$ for Figure 5a, and $a' = 1.03\times10^{-2}$, $b' = 6\times10^{-6}$, $c' = 31$ for Figure 5b. Simultaneously the distance and time scales have been changed when going from Figure 5a to Figure 5b; denoting the variables in the first figure by x, t

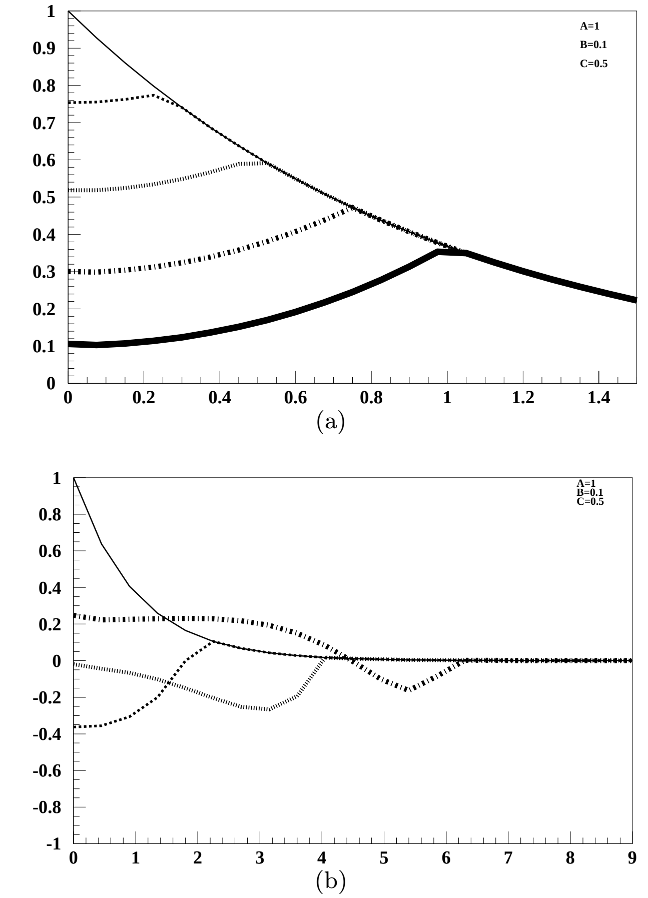

Fig. 6. Space-time correlation function as a function of distance. Horizontal scale: distance. Both charts correspond to the same set of parameters, namely: $a = 1$, $b = 0.1$, $c = 0.5$; (a) gives a detailed view for small values of x and t; from top to bottom, the time-lags are equal to 0, 0.5, 1, 1.5, 2; (b) shows the overall shape of the curves; from top to bottom (on the vertical axis) the time-lags are equal to 0, 4, 8, 12.

and those in the second by x', t' one has the relationship:

$$x = 10^{-2}x'; \qquad t = 0.63t'.$$

This is indeed consistent with the above rule where $\lambda = 10^{-2}, \quad \theta = 0.63$.

3.3 Price correlation as a function of distance

Figure 6a shows the price correlation $\rho_t(x)$ as a function of x for different time lags. The slope for small x (*i.e.* the left hand side of the curves) is first negative for small values of t, then goes through zero and, once positive, becomes larger and larger in agreement with the observation made in Figure 2. Figure 6b provides a more comprehensive picture for larger intervals of variation for both x and t. It shows that the pattern we observed in Figure 6a is only part of the story: beyond $t = 2.5$ the slope and the level of the curves change in an opposite direction. However such curves correspond to large time lags which would in practice be of the order of several years; furthermore the corresponding correlation levels are fairly low (between -0.3 and 0.3) and are therefore difficult to measure.

3.4 Price correlations as a function of time lags

Figure 5a shows the price correlation $\rho_x(t)$ for the same set of parameters than Figure 6a. There are distinctive plateaus for small t; in agreement with observation (Fig. 3) their length increases along with the value of x. In fact it can be shown that the length of a plateau is equal to x/c.

Obviously Figure 5a and Figures 6a, 6b are two different representations for the same function of the variables x and t. For instance, the plateaus in Figure 5a correspond in Figures 6a and 6b to the sections which are common to different curves; thus for $x = 1$ it can be seen in Figure 6a that the curves for $0 < t < 2$ are merged, in other words $\rho_t(x = 1)$ is independent of t; this is indeed consistent with the plateau (0, 2) for the curve $x = 1$ in Figure 5a.

In conclusion one can say that there is a good qualitative agreement between the random wave model and observed properties of the space-time correlation function.

3.5 Order of magnitude of the parameters

Before we perform a systematic adjustment of the model it is of interest to see if we can obtain reliable estimates for some of the parameters from the properties mentioned above.

The parameter a can be obtained fairly easily from the space-like section of the solution corresponding to a zero lag: $\rho_{t=0}(x) = e^{-ax}$. Figure 1a shows that $\rho_{t=0}(x)$ decreases very slowly from 1 to 0.95 which means that ax certainly remains small throughout the whole x range; therefore e^{-ax} can be developed to first order: $\rho_{t=0} \simeq 1 - ax$; in other words, a is the slope of the regression line and its inverse coincides with the so-called correlation length [8]. Thus one obtains: $1/a = 621$ km or $a = 1.6 \times 10^{-4}$ km^{-1}.

The parameter c can be estimated from the length of the plateaus in Figures 3a, 3b, and 3c. This leads to: $c \sim 48$ km/fortnight.

The parameter b could in principle be estimated from the pseudo-period of the correlation function. Yet, this pseudo-period very often turns out to be too long (of the order of about 3 years) to be measured with some accuracy. It should also be mentioned that the correlation function depends upon b in a fairly loose way.

3.6 Parameter adjustment

The curves in Figure 7 provide a comparison of the estimated model to observations. The curve without a plateau is the average intercorrelation for the market pairs Bernay-Evreux and Carcassonne-Toulouse in the period 1825–1841 (the average distance is 64 km). The estimated parameters are: $a = 5.6 \times 10^{-4}, \ b = 3.6 \times 10^{-6}, \ c = 48.$

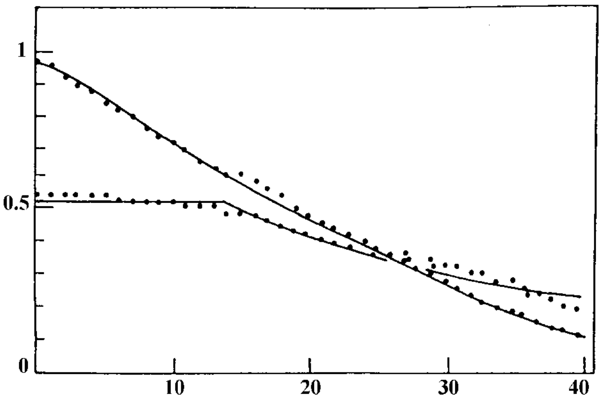

Fig. 7. Comparison of the estimated model to observation. The dots corresponds to the observations. The curves are the theoretical adjustements. The curve without a plateau is the average intercorrelation for the market pairs Bernay-Evreux and Carcassonne-Toulouse (1825–1841), average distance is 64 km. The curve with the plateau is the average intercorrelation for the market pairs Carcassonne-Bernay and Carcassonne-Evreux (1825–1841); average distance is 655 km. The estimated parameters are indicated in the text.

The curve with the plateau is the average intercorrelation for the pairs: Carcassonne-Evreux and Carcassonne-Bernay in the period 1825–1841 (the average distance is 655 km). The estimated parameters are: $a = 9.2 \times 10^{-4}$, $b = 2.6 \times 10^{-5}, c = 48$.

3.7 Economic interpretation

How should the previous wave velocities of the order of 50–100 km/month be interpreted? A first remark is in order about the very notion of price waves. One should not expect the kind of waves that are to be observed when a stone is tossed into a pond because here each market is subject to local disturbances. If one wants to stick to the image of the pond one should rather imagine a pond in which a lot of children are playing. Alternatively one could imagine the waves produced in a pond by drops of heavy rain. In each case the surface of the water would be quite chaotic with small waves moving in every direction. Yet the speed of those waves nonetheless would be gouverned by the laws of hydrodynamics.

Let us now see how we should interpret the order of magnitude of the velocity: 100 km/month corresponds (assuming 12 hours per day) to a speed of 0.3 km/hour; this is clearly slower than any possible means of transportation; even the slowest of them namely transportation by barges is faster than 0.3 km/hour. What we know about the wheat trade in the 19th century provides an explanation.

Before the introduction of railways in the 1860s wheat cargos were rarely shipped over more than 100 km at one time in the center of France. First the wheat was brought by the farmer to the local wheat market (say Bernay for instance) within a radius of less than 40 km of the production place. There it would be bought after a while by a (local) trader and carried over to the nearest provincial capital (say Rouen for instance); from the provincial capital it may eventually go to Paris or into the international trade. At each market place the wheat would have to be stocked until sold. In short, wheat travelled by small steps with more or less long waiting times after each step. A similar process accounts for the propagation of epidemics: a contaminated person A travels, meets and contaminates B; the incubation of B will take several days (or weeks) after which B will be able to contaminate a third person C, and so on. As a matter of fact the evidence shows that the propagation speed of epidemics is strongly correlated with the duration of the incubation period. For instance for the plague the incubation time is of the order of several days and the velocity of the order of 100 km/month, while for Asiatic cholera the incubation interval is less than one day and the velocity is of the order of 800 km/month [2].

Is the wave velocity strongly dependent on the frequency of the fluctuations? The answer is provided by the dispersion relation; if we neglect the small friction term b the latter reads:

$$\omega = c\sqrt{k^2 + a^2}$$

where ω denotes the angular velocity $2\pi/T$ and k the wave number which is related to the wave length λ by $k = 2\pi/\lambda$. The phase velocity is then given by: $v_\phi = \omega/k$ and the group velocity is given by:

$$v_g = \frac{d\omega}{dk} = \frac{kc}{\sqrt{k^2 + a^2}}\,.$$

The phase velocity is always larger than c while the group velocity is always smaller than c; however because a is fairly small, both velocities are in fact not very different from c; three typical values are given in the table below for the following set of parameters $a = 10^{-3}$ km^{-1}, $c =$ 31 km/fortnight:

T [fortnight]	λ [km]	v_ϕ [km/fortnight]	v_g [km/fortnight]
0.17	5.3	31	31
2	63	31	31
72	2387	34	28

The first line corresponds to a frequency of three times a week which was the frequency of the wheat markets in fairly active cities; the last line corresponds to a period of three years. Thus the velocities are substantially different from c only for very low frequencies.

3.8 Evolution of the parameters in the course of time

In a physical system (air, water, copper) the velocity of sound remains constant at least under fixed conditions of temperature and pressure. By contrast the parameters a, b, c characterizing the interactions in the network of markets do not remain constant in the course of time. Both short term fluctuations and a long-term trend can be observed. The following table summarizes some evidence for the parameters a and c.

	France 1825–1841	France 1850–1866	France 1875–1891	United States 1954–1986
$a10^4$ [1/km]	9.0	2.4	6.6	0.18
c [km/month]	70	186	302	–

As could be expected the velocity increases along with technical progress in means of transportation.

4 Conclusion

In this paper we have shown that a time-dependent propagation model based on the spatial arbitrage assumption is able to throw some light on the intriguing evidence displayed by the space-time correlation functions. One should also point out that in its equilibrium formulation that model provides yet other predictions for variables of economic significance, namely for the price differentials between distant markets, for the volatility (*i.e.* the standard deviation) of prices on a single market, and for the trade within a given set of markets. Let us briefly examine to what extent such predictions can be tested against empirical evidence and how such tests could be improved.

(i) In order to improve the testing of the propagation model one would need data for a larger number of markets in a fairly homogeneous region, that is to say a region without ports and without mountains. Such data exist and can be collected for instance at the French National Archives (Paris).

(ii) As far as price differentials are concerned, the model predicts that they should increase with distance at first linearly, then with a diminishing rate for larger distances; such a prediction is indeed confirmed by available evidence (see in this respect [11]).

(iii) As is made clear by Figure 4 spatial arbitrage tends to redistribute available stocks to the markets where supply is insufficient. One should therefore not be surprised that there is a connection between the level of transportation cost and the price volatility on a given market. Unfortunately the price volatility also crucially depends on other factors (such as for instance the elasticity of production with respect to the price level) which have not been taken into account in this model. Nevertheless in earlier periods (from the 17th to the 19th century) where production conditions did not change markedly there is conclusive evidence for a decrease in the volatility along with diminishing transport costs; see in this respect [7,10].

(iv) Spatial arbitrage results in exchanges of cargoes of wheat between different markets. Therefore it is not surprising that the model predicts a specific relationship between average price differentials and the overall volume of trade. Yet, such a prediction is difficult to test empirically for at least two reasons. Since the 19th century wheat production exhibited a strong upward trend while in the model it has implicitly been assumed to be constant on average. Furthermore international trade is limited and biased by a number of factors such as tariffs, importation quotas, bilateral contracts, *etc.* On the other hand domestic interregional trade which would be free of such pitfalls is poorly documented at the statistical level.

So far only limited attention has been given to empirically analyzing the role of distance in economics. While the issue considered in this paper belongs to the field of microeconomics the role of spatial factors is much larger. For instance a number of cultural traits and traditions are of the diffusive type, by which we mean that they are strongly correlated with distance. Such features may have important implications in terms of institutional rules and market organization. In that matter the main difficulty is to find empirical data which permit a *quantitative* description; for an attempt in that direction see [12].

The price data are available upon request from the author.

Appendix A: The stochastic field equation

In this Appendix we derive some properties for the solutions of the model's field equation.

A.1 The solution $\rho(x,t)$

The space-time correlation function $\rho(x,t)$ is defined in the usual way: $\rho(x,t) = c(x,t)/c(0,0)$ with the covariance c being defined by:

$$\begin{aligned} c(x,t) &= \mathrm{Cov}[p(x',t'), p(x'+x, t'+t)] \\ &= E[(p(x',t') - p_m)(p(x'+x,t'+t) - p_m)]. \end{aligned}$$

If, as is assumed here, the process is stationary in time and space, the covariance c only depends upon x and t, and not upon x' and t'.

The stochastic price field $p(x,t)$ satisfies an equation of the form: $L(\partial_x, \partial_t)p(x,t) = N(x,t)$ where L is the differential operator given above in equation (1). Here we are rather interested in the covariance function $c(x,t)$; it satisfies the following equation:

$$L(\partial_x, \partial_t)L(-\partial_x, -\partial_t)c(x,t) = c_N(x,t) \qquad \text{(A.1)}$$

where $c_N(x,t)$ denotes the covariance function of the disturbances $N(x,t)$. The solution of equation (A1) is obtained through Fourier analysis (see [10]); its normalized form $\rho(x,t) = c(x,t)/c(0,0)$ reads (x and t are supposed to be positive):

$$\rho(x,t) = \begin{cases} e^{-ax} & x > ct \\ (-ac)\int_{x/c}^{t} G(x,t')dt' + e^{-ax} & x < ct \end{cases} \qquad \text{(A.2)}$$

where:

$$G(x,t) = e^{-bct^2} J_0\left[\sqrt{(a^2 - b^2c^2)(c^2t^2 - x^2)}\right] Y(ct - x)$$

J_0 is the Bessel function and Y denotes the Heaviside function.

It can be verified that:

$$\lim_{x\to\infty} \rho(x,t) = 0 \qquad \lim_{t\to\infty} \rho(x,t) = 0$$

A.2 Scaling laws

We consider the rescaled variables x' and t' defined by: $x = \lambda x' \quad t = \theta t'$.

Substituting into equation (1) leads to:

$$[(1/c^2\theta^2)\partial^2_{t'^2} + 2(b/\theta)\partial_{t'} + a^2 - (1/\lambda^2)\partial^2_{x'^2}]p = N.$$

Multiplying both sides by λ^2 gives:

$$[(\lambda/\theta)^2(1/c^2)\partial^2_{t'^2} + 2(b\lambda^2/\theta)\partial_{t'} + (a\lambda)^2 - \partial^2_{x'^2}]p = N\lambda^2.$$

This leads to the rescaled parameters:

$$a' = a\lambda \qquad b' = b\lambda^2/\theta \qquad c' = c\theta/\lambda.$$

If we restrict ourselves to the correlation function the scaling factor λ^2 in the right hand side becomes irrelevant because the correlation function does not depend on the variance of the disturbance term.

A.3 Pseudo-period

For practical (for instance estimation) purposes it can be of interest to have an estimate of the pseudo-period of $\rho_x(t)$ for large t; such an estimate can be obtained from the following heuristic reasoning. Equation (A2) shows that $\rho(x,t)$ is the integral of a Bessel function J_0. The latter has a pseudo-period of the order of 6; in the large t region one can approximate $c^2t^2 - x^2$ by c^2t^2; this leads for the function $J_0\left[\sqrt{a^2 - b^2c^2}ct\right]$ to a pseudo-period of the order of: $6/(c\sqrt{a^2 - b^2c^2})$. When tested on numerical examples (see for instance Figs. 5a and 5b) this approximation turns out to be reasonably effective.

Appendix B: Why the interaction between wheat markets can be considered as one of the simplest possible problems in economics

We strongly believe that one of the main contributions econophysics can make to economics is to show what is to be gained by considering problems on account of their simplicity rather than because of their relevance for policy purposes. Let us first develop that point before discussing how it is related to the topic of the present paper.

For most fields there is usually an experiment which provides both a firm foundation and a paradigm for further developments. Galileo experiments (*e.g.* those with a cylinder rolling down a slope) laid the basis of mechanics, Carnot's discussion of a machine using a hot and a cold source was a starting point for thermodynamics, the hydrogen atom provided a key for the understanding of the spectroscopy of more complex atoms, Mendel's experiments on a character determined by two forms of a gene were at the origin of genetics. It can be noted that these problems were all in a sense two-body problems.

In economics the typical two-body problem would be a market with only one producer and one consumer (microeconomics) or an economy with only two sectors (macroeconomics). These cases have indeed received considerable attention as theoretical models. But unfortunately (and in contrast to the problems we mentioned above) no real economic system seems to match such simple assumptions. In other words it was impossible to check the predictions of the basic two-agent model against empirical evidence. As a result such models could hardly provide a firm basis for the investigation of more complicated cases.

In terms of increasing complexity, after the two-body problem comes the N-body problem for identical bodies and a single type of interaction. Is there an economic phenomenon matching those assumptions? The main theme of the present paper was precisely to show that the 19th century wheat markets indeed are a good approximation of such a N-body problem, provided local perturbations are allowed at each marketplace. In other words one of the simplest possible economic system is of a degree of complexity similar to the one- (or, more realistically, the two-) dimensional Ising model with random perturbations.

The next level of complexity would be a N-body problem for non-identical bodies and, as a consequence of the bodies being non-identical, different sorts of interactions. A model of a national economy comprising N sectors clearly belongs to that class. Of course it can be treated in a phenomenological way which is the approach used by macroeconomics, but from a theoretical point of view one should keep in mind that this is already a very difficult problem.

The 4th level of complexity would refer to interactions between N different components each of which consists itself of various sub-components. This is the problem of international economics: interaction between different countries C_i each of which has n_i sectors.

Just for the sake of illustration let us assume that, instead of performing its simple experiments, Galileo had tried to solve a more "practical" problem for instance the rolling movements of a ship. In that case he would perhaps have been able to propose a phenomenological model (restricted of course to one type of ship) but he would certainly not have laid down the foundations of mechanics.

The complexity scale proposed in this appendix can be tested so to say experimentally by going back to the times of Galileo (1564–1642) and of Descartes (1596–1650). The number and the difficulty of the problems Descartes tried to solve in his "Discourse on method, optics, geometry and meteorology" (1637, 1965) [3] is staggering even for the physicist of 1998. As a result the "explanations" he offered most often are only qualitative and completely ad hoc. Interestingly, the only field where he is really successful is optics (*e.g.* the rainbow or the eye) and this is precisely a two-body problem (interaction between light and matter).

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